

On Nil Clean Group Rings

Yuanlin Li

Brock University, St. Catharines, ON, L2S 3A1, Canada

a joint work with

Jian Cui¹ and Haobai Wang²

¹*Anhui Normal University, Wuhu 241002, China*

²*Brock University, St. Catharines, ON, L2S 3A1, Canada*

Abstract

A ring is nil clean if each of its elements is the sum of an idempotent and a nilpotent. In [Sahinkaya, S., Tang, G., Zhou, Y. (2017). Nil-clean group rings. *J. Algebra Appl.* 16(7):1750135.], it was shown that, for a ring R and a symmetric group S_3 , the group ring RS_3 is nil clean iff R and $M_2(R)$ are nil clean. Let D_{2n} be the dihedral group of order $2n$ and Q_{2n} be the generalized quaternion group of order $2n$. In this talk, we investigate a more general question and completely characterize when group rings RD_{2n} and RQ_{2n} are nil clean. It is proved that RD_{2n} is nil clean iff, either $n = 2^k$ and R is nil clean, or $n = 3 \cdot 2^k$ and RS_3 is nil clean, and a similar result is obtained for RQ_{2n} . Furthermore, nil clean group rings with standard involution $*$ are also investigated. We will propose a few research problems in the end of the talk.

Keywords

Nil clean ring; group ring; dihedral group; generalized quaternion group.